Equity Risk Premium Predictability from Cross-Sectoral Downturns*

José Afonso Faias and Juan Arismendi Zambrano

This version: July 2017

Abstract

We illustrate the role of left tail mean (LTM) in equity risk premium (ERP) predictability. LTM measures the average of pairwise left tail dependency among major equity sectors incorporating endogenous shocks that are imperceptible at the aggregate level. LTM, as the variance risk premium, significantly predicts the ERP in- and out-of-sample, which is not the case with the other commonly used predictors. $Ceteris\ paribus$, an increase of two standard deviations in the LTM — in a time-varying disaster-risk consumption-based asset pricing model — causes an increase of 0.70% in the ERP. This paper contributes to the debate on ERP predictability.

Keywords: Predictability, left tail dependence, asset pricing model.

JEL classification: G10, G12, G14.

*Corresponding author: José Afonso Faias, UCP - Católica Lisbon School of Business & Economics, Palma de Cima, 1649-023 Lisboa, Portugal. Phone +351-217270250. E-mail: jfaias@ucp.pt. Juan Arismendi Zambrano, Department of Economics, Finance and Accounting, Maynooth University -National University of Ireland, Maynooth, Ireland. Phone +353-(0)1-7083728. E-mail: JuanCarlos.ArismendiZambrano@mu.ie; ICMA Centre – Henley Business School, University of Reading, Whiteknights, RG6 6BA, Reading, UK. Phone +44-1183788239. E-mail: jarismendi@icmacentre.ac.uk. We thank Rui Albuquerque, Gregory Connor, Adam Farago, Campbell Harvey, Joni Kokkonen, Stefan Nagel, Pedro Santa-Clara, Kenneth J. Singleton, Grigory Vilkov, Andrew Vivian, Jessica A. Wachter, and the participants at the 2016 FMA Annual Meeting, 2016 Research in Options, 2017 FMA European Conference, 2017 Multinational Finance Society Annual Meeting, 2017 European Finance Association Annual Meeting, and the 2017 Foro Finanzas for helpful comments and discussions. We are particularly grateful to João Monteiro, Pavel Onyshchenko and Duarte Alves Ribeiro for outstanding research assistance. This research was funded by grants UID/GES/00407/2013 and PTDC/IIM-FIN/2977/2014 of the Portuguese Foundation for Science and Technology-FCT.

1. Introduction

Rare events, such as the 2007-2009 global financial crisis, are crucial in asset pricing. Rietz (1988) introduces a disaster-risk-based model to explain the equity premium puzzle. In the subsequent literature, Barro (2006) broadens this model to several countries, and Wachter (2013) shows that investors' perceptions of risk change when rare events occur. If all these models display large conditional equity premia, then the challenge is finding conditional information that best captures this disaster risk and its implied predictability. In general, aggregate-level variables are usually used to predict asset returns. However, seeking and discovering new relations using the non-aggregated quantities of the aggregated phenomenon is intuitive. Indeed, if endogenous sectoral shocks hold specific information, they should be used to reflect uncertainty in asset prices. This is even more important when addressing tail comovement, since at the aggregate level tail risk is partially diversified away. Is there a benefit to incorporating endogenous sectoral tail shocks when predicting asset returns?

On the one hand, one way to incorporate rare events in finance is to use extreme value theory (e.g., Longin and Solnik 2001, Bae, Karolyi, and Stulz 2003, and Hartmann, Straetmans, and de Vries 2004). For example, Poon, Rockinger, and Tawn (2004) advocate the use of risk measures based on extreme value theory rather than traditional risk measures, such as volatility or value-at-risk. They demonstrate that the latter are unsuitable for measuring tail risk, which may lead to inaccurate portfolio risk assessment. On the other hand, researchers have shown that a powerful solution when examining aggregate-level variables is the use of sectoral information, because different shocks can be recognized at the sectoral level but are invisible at the aggregate level (e.g., Horvath 2000, Veldkamp and Wolfers 2007, Comin and Mulani 2009, and Holly and Petrella 2012). For example, Hong, Torous, and Valkanov (2007) show that industry interdependencies are essential for the predictability of market returns. This paper provides a positive answer to the earlier question. We are the first to analyze the joint effect of tail risk and endogenous sector heterogeneity to predict asset returns.

Our first main contribution is to define a new simple and tractable measure of a country's

⁻

¹ A more straightforward example can be found in the field of natural sciences, e.g., aggregated behaviors can hide information about non-aggregated pieces, just as studying human cells will give us information that is not perceptible through the study of the human body as a whole.

left tail dependency, which has strong and significant predictive power for the U.S. equity premium in-sample (IS) and out-of-sample (OOS). Based on extreme value theory, we first compute the bivariate sectoral tail dependence for each pair of sectors in a country to measure the joint extreme events between the two sectors.² Then, we compute the average tail dependence between sectors within a country. We designate this average value by the left tail mean, LTM. The main intuition is that existing aggregate market tail measures average out important information about tail risk in the economy, while average tail dependency among sectors conveys this information more precisely. In our setting, out-of-sample equity risk premium (ERP) predictability by the LTM is the result of an optimal hedging strategy³ in which the investor is searching for the "timing" of rare consumption disasters, which have a substantial impact on their equity assets. Investors first observe the aggregate variable; then, a market sectoral joint downturn movement is a strong signal that a systemic event is under way. LTM is a good descriptor of endogenous sectoral tail dependency, and an increase in sectoral tail dependency precedes a disaster. In a setting that assumes no disaster events, a sudden increase of endogenous sectoral tail dependence (LTM increases) will push investors to anticipate a disaster and therefore to rebalance all their positions from equity holdings to other assets (e.g., treasuries) in a typical flight to quality behavior. This process will reinforce the increased value in the observed LTM that will eventually stop either when investors realize they are not in a disaster event or when the disaster occurs with all sectors experiencing a downfall that is not necessarily of the same magnitude across sectors but that has the same starting point. The predictability of a similar "fear" behavior is also observed in Bollerslev, Todorov and Xu (2015). We also compute four other measures of dependence: RTM, CORR, ALTM, and SLTM. The right tail mean, RTM, and the correlation sectors' mean, CORR, are computed as the LTM but for the joint right tails and joint Pearson correlations, respectively. ⁴ The ALTM is the univariate market tail risk, and the SLTM is the average univariate sectors' tail risk. We show that the level of the LTM is time-

² Other authors (e.g., Patton 2009) use Copula functions to model dependence structure. Hilal, Poon, and Tawn (2011) argue that the copula approach imposes conditions on the dependence structure that are too rigid and that the validity of its assumptions was not tested. However, some of the foundations in the extreme value theory are built on the Copula approach, though they impose looser restrictions in the distributions used.

³ In the online appendix, an asset management exercise is provided with the optimal hedging strategy of an investor that consider the existence of rare disasters, and that measures tail dependence with *LTM*.

⁴ The *CORR* intrinsically assumes normality of the truncated distribution of returns.

varying, quite adaptive, and it dominates the levels of the *RTM*, *CORR*, *ALTM*, and *SLTM* through time. It also reacts quickly and more strongly than the other measures. This is evidence that (1) returns in the tails are not drawn from a normal distribution, (2) the tails are asymmetric, and (3) it is important to study the link between the sectors rather than only the risk of each sector or only the overall market.⁵

A long dispute about the predictability of several common variables (e.g., Campbell and Thompson 2008, Goyal and Welch 2008, Rapach et al. 2010, Ferreira and Santa-Clara 2011, and Li et al. 2015) has persisted. We participate in this debate. We run predictive regressions as in Goyal and Welch (2008). Using a comprehensive set of common variables, we show that there are only two predictors that offer both in- and out-of-sample significant, higher predictive power than the historical average of the equity premium. These two predictors are the LTM and the variance risk premium. Their static and time-varying performances are similar, although their unconditional correlation is quite low, 0.04, indicating a different but valuable impact of these two predictors. We select the new proposed dependence variables as predictors alongside the usual variables, including the short interest index, the variance risk premium, the dividend-price ratio, and the detrended Treasury bill rate. Although the short interest index has in-sample predictability, it clearly fails out-of-sample. We also show that ERP predictability from LTM is due to the sectors' joint shocks. There is no such predictability in the univariate left tail risk of the aggregate market or in the average of the univariate left tail risk of individual sectors. In fact, using fewer sectors to compute LTM results in lower predictability. We also present evidence that not all sectors and their left tail joint dependencies are related to future risks in the same way. Nevertheless, using a value-weighted average in LTM by the size of each sector leads to the same qualitative conclusions. All these results support our view that the interdependencies of joint left tail sector shocks are an important source of predictability. Additional robustness tests include time-varying regressions (Dangl and Halling 2012), stock return decompositions (Rapach et al. 2016), and the study of predictability during business cycle recession periods (Henkel, Martin, and Nardari 2011).

Our second main contribution is to provide an endogenous sectoral asset pricing model

⁵ This is in line with studies such as Ang and Chen (2002), who find an asymmetry in their dependence structure that is 12% larger in negative events than the correlation implied by the normal distribution, whereas there are no significant differences in the dependence structure for positive events.

that values bivariate tail dependency effects between equity assets. The benefit of this sectoral model is that the cross-sectional information helps triangulate time-varying disaster-risk, as in Kelly and Jiang (2014). This endogenous sectoral model is the result of a growing literature on tail dependency (Longin and Solnik 2000, Ang and Chen 2002, and Poon, Rockinger and Tawn, 2004) considering that comovements in sector consumption and sector equity prices have an impact on the equity risk premium (ERP). The endogenous sectoral model is a simple extension of the univariate rare disaster consumption models: it preserves the properties of univariate models with respect to their equilibrium while disentangling the endogenous statistical properties of the variables. Our sectoral model extends the literature on rare disaster consumption models (Rietz 1988, Barro 2006, and Wachter 2013) to a multi-asset consumption model in which the aggregated market consumption is the result of the aggregated sectors' consumption.⁶ Thus, the probability of a rare disaster is directly linked to the left tail dependence and therefore predicts the ERP in-sample. To test this prediction with real data, we use the previous measure of a country's stock market tail dependence that includes within-country pairwise sector tail dependences. Ceteris paribus, we find that a 17% increase in the average bivariate left tail dependency (LTM) drives an absolute increase of the ERP by at least 0.63% (23% in relative terms). This increase of 17% is realistic since it has been observed in a monthly time series. In a different setup, recent evidence demonstrates the importance of left tail dependence. Chabi-Yo, Ruenzi, and Weigert (2013) show that investors require a premium to hold portfolios with high left tail dependence as insurance against negative extreme events.

In the classic formulation of the *ERP* puzzle, Mehra and Prescott (1985) recognize that for an Arrow-Debreu economy, the difference between equity and Treasury bill returns was excessively large, implying that they can explain the large equity premium only when considering frictions in the economy. Nevertheless, recent evidence from rare disaster models, such as Barro (2006), proposes that the puzzle is solved in a frictionless economy when large consumption drawdowns are included in a model.⁷ Our endogenous sectoral model strengthens

⁶ Aggregate consumption is a linear function of sectoral consumption; however, non-linear effects from the endogenous sectors' interaction that correspond to the proximity to a rare-disaster event generate a positive consumption effect reflected in the time-varying *ERP*.

⁷ In a recent consumption model proposed by Martin (2013), a multi-asset extension of the Lucas (1978) tree, the price-dividend ratio dynamics are a complex result of the multi-asset consumption and the multivariate dividend factors. Although it may be seen as a natural model selection for proposing multi-asset pricing consumption models,

the idea that equity premiums are predictable, and this predictability is associated with the proximity of a consumption disaster.⁸ In the resulting model, the probability of a disaster is linked to the time-varying dependency of economic sectors: consumption and equity sectors are linked as in the rare disaster models of Barro (2006) and Wachter (2013) using a consumption/dividend relation. We use this link function to establish the relationship between the empirical results and the theoretical sector consumption model. To distinguish the behavior of consumption returns in times of normalcy from that in times of disaster, we apply a method of moments, as in the multi-asset model for the systemic risk of international portfolios in Das and Uppal (2004). The endogenous sectoral *ERP* predictability is supported not only in the rare disaster consumption literature but also in the classic puzzle literature; the existence of a strong linear relation (R-squared greater than 19%) between the marginal utility of consumption and the *LTM* of the sector returns from January 1993 to December 2013 is a sign of the time-varying relation in the Mehra and Prescott (1985) classic puzzle model in which the *ERP* is the product of the covariance of the marginal utility and the equity returns.⁹

Finally, the paper's most natural point of comparison is to the work of Wachter (2013), who includes no endogenous sectors in her model. In our theoretical model, we use the Wachter (2013) time-varying rare disaster model's calibrated parameters for the probability of disasters, and we compute the *ERP* for different values of the *LTM*. We show a clear improvement of using an endogenous sectoral model over the non-sectoral model, theoretical and empirically. Wachter (2013) extracts an implied disaster-risk measure based on simulations. In our case, we provide a direct, easy, and tractable measure, *LTM*, which strongly predicts the equity risk premium in- and out-of-sample. There is an indirect route to check the natural improvement of our endogenous sectoral model. The measure in Wachter (2003) designated by implied disaster probability (*IDP*) is implied from roughly the smoothed earnings-price ratio. If there is no

our endogenous sector modeling preserves the simplicity of the equilibrium with univariate consumption models while allowing exploitation of the internal statistical properties of the variables that (in our view) generate *ERP* predictability.

⁸ Hansen and Singleton (1983) study the restrictions on the modeling of the joint distribution of consumption and asset returns. In their modeling, they found that when the consumption is log-normally distributed by a random walk, the asset returns will be serially uncorrelated. However, asset returns will have predictable components when consumption growth has "nontrivial predictable" components.

⁹ This is a strong correlation value, as when we use the *TBILL* alone to explain marginal utility, the IS R-squared is only 7.68%. However, when tested jointly with the *LTM*, the IS R-squared increases to 21%.

predictability on this ratio or its orthogonal component to *IDP*, there is a substantial probability that the implied disaster probability will not have such predictability. We show that in our time span, the earnings-price ratio or smoothed earnings-price ratio has no significant predictability of the equity risk premium in- and out-of-sample. In truth, *IDP* has no predictability over the *ERP* in this time span (between 1993 and 2010). It is important to stress that *LTM* has a low correlation with *IDP*. Therefore, this paper shows that endogenous sectoral considerations lead to a better empirical measurement of time-varying disaster risk than a model with no such considerations.

The remainder of the paper is organized as follows. In Section 2, we explain the data used and how to compute the dependence variables. Section 3 presents the methodology and the results of the predictability exercise. In Section 4, we discuss the theoretical motivation. Finally, the paper closes.

2. Data

The main analysis uses U.S. end-of-month observations starting in January 1993 and ending in December 2013 since dependence variables require 20 years of data to initialize (i.e., we use data starting in January 1973 to initialize the dependence variables). This analysis period is analogous to many other papers, such as Rapach et al (2016). We first describe how to compute the tail dependence variables. Then, we explain the traditional predictors used in past literature and their relation with the dependence variables.

2.1. Tail Dependence Variables

Extreme value theory (EVT) is used to estimate bivariate tail distribution. Considering that only the dependence structure is important in this analysis, we exclude the marginal distributions of this setting. Following Poon, Rockinger, and Tawn (2004), the bivariate returns (X,Y) are transformed into unit Fréchet marginals (S,T)

$$S = -\frac{1}{\log F_X(X)} \text{ and } T = -\frac{1}{\log F_Y(Y)},\tag{1}$$

where F_X and F_Y are the respective marginal distribution functions for X and Y. Poon, Rockinger, and Tawn (2004) define the tail dependence measure as

$$\overline{\chi} = \lim_{s \to \infty} \frac{2\log \Pr(S > s)}{\log \Pr(S > s, T > s)} - 1,$$
(2)

where $-1 < \overline{\chi} < 1$. This method accurately captures the asymptotic independence, as $\Pr(S > s \mid T > s) \to 0$. This measure has the clear advantage of being interpreted as the correlation coefficient. Values of $\overline{\chi} > 0$, $\overline{\chi} = 0$, and $\overline{\chi} < 0$ loosely correspond to when (S,T) are positively associated in the extremes, exactly independent, and negatively associated, respectively. Poon, Rockinger, and Tawn (2004) show that $\overline{\chi}$ is the correlation coefficient in the case of the bivariate Gaussian dependence structure.

Next, we define $Z = \min(S,T)$ and rank all its values from $Z_{(1)}$ to $Z_{(n)}$. The maximum likelihood estimator is given by

$$\hat{\overline{\chi}} = \frac{2}{n_n} \left(\sum_{j=1}^{n_n} \log \left(\frac{z_{(j)}}{u} \right) \right) - 1, \tag{3}$$

where n_u is the number of observations above the threshold u. Throughout this paper, n_u is 5% of n. We interpret this variable as the average log excess returns relative to the threshold u. This is similar to the notion of expected shortfall, but instead of considering the expected return values above a threshold – value-at-risk in this case – our variable uses the expected log returns in excess of a threshold value. This implies that the variable is much more stable through time since we study the distance of each extremal observation from a percentile, rather than studying a censored distribution.

Traditionally, univariate distributions are used to build a time series measure of tail dependence for a country (Kelly and Jiang 2014, Poon, Rockinger, and Tawn 2004, and Chabi-Yo, Ruenzi, and Weigert 2013). However, these measures do not capture all aspects of the tail dependence. Several papers show that industry interdependencies are important in predictability (Hong, Torous and Valkanov 2007, Cohen and Frazzini 2008, Menzly and Ozbas 2010, and Rapach et al. 2015). Therefore, one can use information from the different sectors of a country to obtain a more complete picture of that country.

We define a new and simple measure of a country's tail dependence by combining the

 $^{^{10}}$ Weak assumptions are required to estimate $\overline{\chi}$ and are specified in Poon, Rockinger, and Tawn (2004).

¹¹ Longin and Solnik (2001) use bootstrapping to define the optimal threshold level for several large economies. They find that on average, a level of 4-5% of the total number of observations should be considered as a threshold. We also considered other values of n_u , such as 10% and 20%. In these cases, *ERP* predictability is achieved but is smaller, confirming the importance of considering tail values.

information from all intra-country tail dependences between the sectors. First, $\bar{\chi}$ is computed for all pairs of sectors within a country using weekly returns and a rolling window of 1,040 weeks (20 years). This computation is performed for the two tails of the bivariate distribution, the positive (negative) extreme joint events considered to be the right (left) tail. We censor the values of estimated $\bar{\chi}$ between -1 and 1. Then, a cross-section arithmetic mean of $\bar{\chi}$ for all pairs of industries within a country is computed for each of the tails. This is a similar procedure to the one used by Rapach et al. (2010) to aggregate different estimates to forecast returns. They argue that the equally weighted aggregation shows stronger performance in practice than other sophisticated weighting systems.

The cross-section measure for the left tail is the *LTM*, and is given by

$$LTM_t = \binom{n}{2}^{-1} \sum_{i,j} \overline{\chi}_{i,j,t'}^L \tag{4}$$

where $\overline{\chi}_{i,j,t}^L$ is the left tail risk measure for each pair of sectors i and j at time t, where n is the number of sectors in the country.

The cross-section measure for the right tail is the *RTM*, and is given by

$$RTM_t = \binom{n}{2}^{-1} \sum_{i,j} \overline{\chi}_{i,j,t}^R, \tag{5}$$

where $\bar{\chi}_{i,j,t}^R$ is the right tail risk measure for each pair of sectors i and j at time t, where n is the number of sectors in the country.

As a benchmark, we also compute the same type of measure using the traditional Pearson correlations. The Pearson correlation measures the average of deviations from the mean without making any distinction between negative and positive returns. The cross-section measure for the Pearson correlation is designated by the *CORR* and is given by

$$CORR_t = \binom{n}{2}^{-1} \sum_{i,j} \rho_{i,j,t'} \tag{6}$$

where $\rho_{i,j,t}$ is the Pearson correlation measure for each pair of sectors i and j at time t, where n is the number of sectors in the country.

Additionally, we consider two univariate tail risk variables. The first one is the

¹² This somewhat large number of observations is required since the tail dependence measure uses only 5% of the total number, which corresponds to 52 observations, a sample size that is usually assumed to be a large sample for inference.

aggregated market univariate measure for the left tail (ALTM) and is given by

$$ALTM_t = \overline{\chi}_{M,t}^L, \tag{7}$$

where $\bar{\chi}_{M,t}^L$ is the univariate left tail risk measure for the market at time t. The second measure is the univariate sectors' left tail mean (*SLTM*) and is given by

$$SLTM_t = \frac{1}{n} \sum_i \bar{\chi}_{i,i,t}^L, \tag{8}$$

where $\overline{\chi}_{i,i,t}^L$ is the univariate left tail risk measure for each sector at time t, where n is the number of sectors in the country.

Sector data at the weekly level is used to construct the dependence variables. The Friday closing price is considered for each of the target indices. The 10 selected sectors are the following: oil & gas (OIL), utilities (UTIL), financial (FIN), technological (TECH), consumer goods (CG), basic materials (BM), healthcare (HC), industrials (IND), consumer services (CS), and telecommunications (TLC). The data are obtained from Thompson Datastream and span from January 1, 1973 to December 30, 2013. Weekly frequency is preferred over monthly and daily. Hartmann, Straetmans, and de Vries (2004) also make a similar choice of frequency to study tail dependence. The choice of weekly frequency rather than daily avoids the problems of non-synchronous trading and heteroskedasticity, which affect the estimates of tail dependence (Poon, Rockinger, and Tawn 2004). The choice of weekly observations rather than monthly implies a fourfold increase in sample size, which is important in this setting. Weekly frequency is used in the time series measures. However, because predictability is performed monthly, the variables had to be converted to a monthly frequency. Here, the monthly measure is the average of weekly values within each month. In the content of the problems of the pr

Panel A of Table 1 presents the descriptive statistics of the five dependence variables: LTM, RTM, CORR, ALTM, and ALTM. Figure 1 presents their evolution. Panel A presents the levels and Panel B presents the standardized variables. The standardization uses the first two unconditional moments. All variables are quite persistent, which would be expected by their definition. However, this high serial correlation is also standard in the traditional predictor variables. As expected, the LTM dominates the CORR over the entire period. In a different setup,

¹³ Ten sectors correspond to 45 pairs in computing *LTM*, *RTM*, and *CORR*.

¹⁴ We also use the last weekly observation of each month, and the results are of similar magnitude. They are available upon request.

Ang and Chen (2002) also find that negative tails deviate more from the normal distribution than right tails. The *RTM* predominantly lies between the *LTM* and *CORR* measures. Additionally, the distance between the three variables is time-varying with the *RTM* closer to the *CORR* coefficient in normal periods and closer to the *LTM* in periods of financial crisis (see the shaded area in Figure 1). This is related to higher volatility during crisis periods, which leads to positive and negative joint extremes. Finally, the *CORR* coefficient has the smoothest pattern of the five variables. Although the levels are quite persistent, what matters for the predictability exercise is the standardized variables. When examining the standardized variables, *LTM* reacts more strongly, and it is quite adaptive in several episodes, such as in the periods between 2001 and 2002 and between 2008 and 2009. *LTM* clearly deviates quite often from the other four dependence variables. Notably, the univariate measures, *ALTM* and *SLTM*, are almost flat after 2011, which reveals their inadequacy in capturing changes in the *ERP*.

[Insert Figure 1 here]

2.2. Other Predictor Variables and Equity Risk Premium

We use traditional predictors and the new proposed dependence variables to study the predictability of the stock market equity premium. All variables lag the stock market equity premium by one month. At the start of each month, the investor can choose from 20 variables. The set of traditional variables are the common variables used in the literature (e.g., Goyal and Welch 2008) that are related to stock market characteristics, interest rates, and broad macroeconomic indicators. The default spread (*DFS*) is the difference between the returns of BAA-rated and AAA-rated bonds. The term spread (*TMS*) is the difference between long-term bond returns (10-year) and T-bill returns. The dividend-price (*DP*) ratio is defined as the difference between the log of the 12-month moving sum of dividends paid on the S&P 500 Index and the log of prices. The detrended T-bill (*TBILL*) rate is the T-bill rate reduced by the 12-month backward moving average. The book-to-market (*BM*) ratio is the book-to-market ratio of the Dow Jones Industrial Average. Dividend yield (*DY*) is the difference between the log of the 12-month moving sum of dividends paid on the S&P 500 Index and the log of lagged prices. The

¹⁵ The two shaded areas in Figure 1 correspond to the two recessionary periods, as defined by NBER (http://www.nber.org/cycles.html). The starting period is the peak, and the ending period is the trough for real GDP in the U.S.

dividend payout (DE) is the difference between the log of the 12-month moving sum of dividends paid on the S&P 500 Index and the log of the 12-month moving sum of earnings on the S&P 500 Index. The earnings-price (EP) ratio is the difference between the log of the 12month moving sum of earnings on the S&P 500 Index and the log of prices. The realized stock variance (SV) is the sum of squared daily returns on the S&P 500 Index during a month. Net equity expansion (NTIS) is the ratio of the 12-month moving sums of net equity issues by NYSE listed stocks to the total end-of-year market capitalization of NYSE stocks. Inflation (INFL) is the Consumer Price Index provided by the Bureau of Labor Statistics. The long-term yield (LTY) is the long-term U.S. government bond yield. The variance risk premium (VRP) is the difference between the expected 1-month ahead stock return variance under the risk-neutral measure and the expected 1-month ahead variance under the physical measure. The short interest index (SII) is computed as the standardized linear detrended log of the equal-weighted mean of short interest (as a percentage of shares outstanding) across all publicly listed stocks on U.S. exchanges. The cross-sectional tail risk (CSTR) is the cross-sectional tail risk measure from Kelly and Jiang (2014). It is constructed by applying the Hill estimator to the whole NYSE/AMEX/NASDAQ cross-section (share codes 10 and 11) of daily returns within a given month. We compute this variable. The VRP is obtained from Hao Zhou's website and the SII is obtained from David Rapach's website. The remaining variables are obtained from Amit Goyal's website. These variables are from December 1992 to December 2013 since dependence measures did not start until this point.

[Insert Table 1 here]

The summary statistics are presented in Panel B of Table 1. All summary statistics are generally consistent with the literature. Many of these economic variables often exhibit near-unit-root persistence. Table 2 presents the correlation matrix of these variables and the dependence variables. The findings reveal some typical connections between the variables. *DY* and *DP* have a strong and positive correlation of 0.99. *DE* and *EP* have a strong and negative correlation of -0.83. The *BM* and *DP* have a moderate and positive correlation of 0.67. Note that *DFS*, *TMS*, *DP*, and *TBILL* are weakly correlated with a maximum absolute value of correlation of 0.42. *DFS* and *DY* have a correlation greater than 0.40 with many variables. It is remarkable that *INFL* and the *VRP* are weakly correlated with any of the other variables. The dependence variables are strongly unconditionally correlated: the correlation between the *RTM* and the

CORR is 0.82; between the LTM and the CORR is 0.74; and between the LTM and the RTM is 0.59. It is remarkable to see that the LTM strongly correlates with NTIS and it is somewhat strongly correlated to DP, TBILL, and DY. In untabulated results, we also compute the contemporaneous 24-month rolling window correlation between the LTM and each of these variables. There is a remarkable volatile movement in the contemporaneous correlation between LTM and many variables that even shows quite a few sign changes, which turn out to be significant, and there are moments in time with a correlation of zero, although most of the time, the correlation is of course positive and significant. As an example, we analyze the case of NTIS. Notably, the correlation achieves values of -0.80 in 1997, 1999, and 2005. For DP, the correlation is below -0.70 in 2005, 2008, 2011, and 2012. Thus, a broad selection of effects is captured. The RTM is strongly correlated to the CORR and somewhat related to NTIS. The CORR is strongly related to NTIS and somewhat related to DY and LTY. This shows that all these dependence variables actually capture different effects from the economy.

The equity premium is simply the difference between the stock market returns and the short-term rate. The U.S. MSCI index, which is retrieved from Thompson Reuters Datastream, is used as a proxy for the stock market. The short-term bond is proxied by the 3-month U.S. T-bill and is obtained from the Federal Reserve Economic Data (FRED). Panel C of Table 1 presents the summary statistics for the equity risk premium for the period from January 1993 to December 2013, which are well known and similar to the previous literature.

[Insert Table 2 here]

3. Predictability

In this section, we are interested in testing the predictability of the *ERP* using *LTM*. We care about both in- and out-of-sample results. Then, we contrast these results with the ones using the traditional predictors and other measures of tail dependence. Next, we demonstrate that *ERP* predictability is robust to different definitions of the *LTM*. At the end of this section, we also present the incremental value of the *LTM* by allowing its combination with other predictors.

3.1. Methodology

We apply the widely used methodology of comparing the sum of squared errors (SSE) of the predictive regression with the SSE of the average historical equity risk premium (e.g., Goyal and

Welch 2008, Ferreira and Santa-Clara 2011, Campbell and Thompson 2008, Rapach *et al.* 2010, and Li *et al.* 2015).

First, we obtain in-sample (IS) results. We run a predictive regression for the entire sample of available data in the following form:

$$ERP_t = \alpha + \beta x_{t-1} + \varepsilon_t, \tag{9}$$

where x_{t-1} is the predictor at time t-l and ERP_t is the equity risk premium at time t. Then, we compute the R-squared of this regression as

$$R_{IS}^2 = 1 - \frac{\sum_{t=2}^{T} (ERP_t - \widehat{ERP}_t)^2}{\sum_{t=2}^{T} (ERP_t - \overline{ERP}_t)^2}$$

$$\tag{10}$$

where T is the size of the sample, \widehat{ERP}_t is the predicted value from Equation (9) and \overline{ERP}_t is the sample average of the risk premium using an expanding window until time t. If the R-squared is positive, then the predictor forecasts the value of the equity risk premium better than the historical risk premium average. As the R-squared increases, the quality of the forecast improves.

We also evaluate the out-of-sample (OOS) predictive power, which is closer to real-time forecasting. To predict the value of the risk premium OOS at time t+1, we only use the data available until time t instead of the entire available sample. Hence, the regression is re-estimated before every prediction. The OOS R-squared is given by

$$R_{OOS}^2 = 1 - \frac{\sum_{t=m+1}^{T} (ERP_t - \overline{ERP}_t)^2}{\sum_{t=m+1}^{T} (ERP_t - \overline{ERP}_t)^2}$$

$$\tag{11}$$

For the OOS forecast, we require m periods for the initial estimation period for the first prediction, and we then either roll over the estimation period (rolling window) or expand it for the next forecasts (recursive or expanding window), allowing us to obtain q = T - m OOS observations. Consistent with Goyal and Welch (2008), we use an expanding window with an initial estimation period of five years. To test the statistical significance of IS and OOS predictions, we use the Clark and West (2007) test of equal forecast ability. The test helps to identify whether the Mean Squared Percentage Errors (MSPE) of prediction is significantly lower than MSPE of the historical equity risk premium average. In practice, this is identical to

testing the null hypothesis of $R_{OOS}^2 \le 0$ against the alternative hypothesis of $R_{OOS}^2 > 0$. We apply Hodrick's (1992) standard error correction for overlapping data using 12 lags.¹⁶

3.2. Results

Panel A of Table 3 presents the in- and out-of-sample results. All predictors present positive insample R-squared, although only a few are statistically significant: TBILL, SV, VRP, CSTR, SII, CORR, RTM, and LTM. Notably, all our bivariate dependence measures have positive and significant R-squared. The VRP has the highest (6.13%) and the LTM the second highest (4.91%). Next, we evaluate the out-of-sample predictability. As expected, most of the predictors exhibit a significant reduction in R-squared and lose significance when compared to the insample results. The only variables with positive and significant results are the VRP and LTM. The out-of-sample R-squared values are 4.79% and 2.94%. Both are considered very high levels of predictability. Harvey et al. (2016) claim that given extensive data mining in the current literature, it does not make any economic or statistical sense to use the usual significance criteria for a newly discovered factor, e.g., a t-ratio greater than 2. Instead, they suggest that a newly variable needs to clear a much higher hurdle, with a t-ratio greater than 3.0. We investigate the statistic value of the slope of the predictive regression and the R-squared statistic for LTM. The tstatistic of the IS slope of the predictive regression is 3.59 and the R-squared statistic is 4.61, which is clear evidence that this is a significant effect. We check on unreported results that much of this predictability is derived from recession periods, as one would expect. The results also support the view that valuation ratios have lost their predictive power over time.

[Insert Table 3 here]

Next, Panel B of Table 3 presents the in- and out-of-sample R-squared for different alternative specifications of the *LTM*. First, we present the results for the aggregate measure of left tail risk, *ALTM*. There is no in- or out-of-sample predictability by the univariate market left tail risk. The IS R-squared is 0.15% and not statistically significant, and the OOS R-squared is negative, -2.57%. This is evidence that seeing the shocks at the sector level is important. We

.

¹⁶ Richardson and Smith (1991) argue that overlapping return observations produce a moving average structure in the errors of the forecast, hence jeopardizing the reliability of the tests based on Ordinary Least Squares (OLS) and even Newey-West (1987) standard errors. According to Ang and Bekaert (2007), Hodrick's (1992) standard error correction yields the most conservative test results.

also compute the measure using univariate left tail risk for each sector, SLTM. The IS R-squared is 0.17% and again not statistically significant, and the OOS R-squared is -1.86%. The conclusion is the same: there is no predictability using this variable. This demonstrates that joint sectoral shocks, i.e., their interdependencies in the tails, are the most important factor, not shocks to individual sectors. We then disaggregate the *LTM* to each sector contribution. We compute the joint left tail risk measure for all the pairs that contain a specific sector and average these 9 pairs so that we get the LTM for each sector. All sectors present positive IS R-squared, and many present significant results: BM, IND, HC, CS, TLC, FIN, and UTIL. However, only the sectors HC, CS, and FIN present positive and significant OOS R-squared. Sectors CS and FIN even present higher R-squared than the LTM, but we prefer to use the LTM measure as a conservative choice. A way to incorporate the importance of each sector (composition effects) through time is to consider their average size at each point in time. Thus, we construct the variable LTM using a value-weighted average rather than an arithmetic average. The in-sample R-squared is positive and statistically significant, 5.21%, and the out-of-sample R-squared is positive and statistically significant, 3.98%. These results are even stronger and show that the measure using a sector's relative importance plays a stronger role. We also investigate the role of having less or more sectors in the definition of LTM. In Panel B of Table 3, we present the results for 5, 10, 17 and 38 sectors using Fama-French industry classifications. As expected, there is no predictability when using a small number of sectors. For 5 sectors, the IS R-squared is only 0.38% and the OOS R-squared is -2.32%. For 10 sectors (the same baseline measure but different data), the numbers are 5.61% and 3.24%, respectively. ¹⁷ For 17 sectors, the numbers are 6.67% and 5.11%, respectively. For 38 sectors, the numbers are 6.75% and 5.14%, respectively, which is clear evidence that increasing the number of sectors improves predictability results. Nevertheless, we keep the initial version of the LTM as a conservative choice. In unreported results, we use the median and the 95% truncated mean as in Rapach, Strauss and Zhou (2010). The results are qualitatively the same.

To sum up, all these results show that using a variable with a joint left tail sectoral shock is very important for predictability, but the predictability is stronger when considering all sectors simultaneously, as in the *LTM*.

¹⁷ We keep the current dataset using MSCI data as a conservative option.

Finally, we aim to understand the incremental predictability value of the *LTM*. Therefore, we compare univariate against bivariate predictability for each of the 19 variables (excluding *LTM*). In the case of bivariate predictive regressions, we combine the *LTM* with each one of the 19 alternative predictors. As expected (due to an additional variable), the IS R-squared increases for all variables. More notable is the fact that the OOS R-squared also increases for all predictive regressions. This can be confirmed in Figure 2. There is a northeast shift of all the observations in the plot. For example, when combining the *LTM* with the *VRP*, the IS R-squared improves from 6.13% (univariate regression) to 10.68% (bivariate regression), and the OOS R-squared moves from 4.79% (univariate regression) to 7.26% (bivariate regression). This is surprising since it is common than when one adds an additional predictor in the predictive regression, the IS results would improve but OOS results would drop due to an increased estimation error. For all of the predictors used, on average, the IS R-squared increases by 4.22 p.p. and the OOS R-squared increases by 2.37 p.p. To sum up, the *LTM* is not only able to predict the *ERP* on its own, but it also improves the predictability of each of the traditional variables.

[Insert Figure 2 here]

Wachter (2013) shows that a continuous-time endowment model in which there is a time-varying risk of a rare disaster can explain the equity premium without assuming a high value of risk aversion. This model, however, has no endogenous sectors and uses an implied disaster-risk measure based on simulations designated by implied disaster probability (IDP). We provide a direct, easy, and tractable measure, *LTM*. We check the predictability of *ERP* by *IDP* in our time span until 2010. There is no such predictability. The in-sample R-squared is 0.09% and the out-of-sample R-squared is -2.85%. Both are not significantly different from zero for a significance level of 5%. Wachter's measure is implied from roughly the actual earnings-price ratio. In our time span, the earnings-price ratio has no significant predictability in- and out-of-sample as seen in Panel A of Table 3. In fact, Wachter (2013) uses the smoothed earnings-price ratio from Shiller (1989). The correlation between *IDP* and the smoothed earnings-price ratio is -0.68. This is a strong negative value. We also run the predictability regressions using this ratio. The in-sample R-squared is 0.69% and the out-of-sample R-squared is -1.33%. Both are not

¹⁸ We thank Jessica Wachter for providing these data, which are only available until 2010.

significantly different from zero for a significance level of 5%. We also get the orthogonal component of IDP from the smoothed earnings-price ratio by computing the residuals of the former on the later variable. Even the residuals cannot predict the *ERP*. The in-sample R-squared is 0.22% and the out-of-sample R-squared is -2.38% and both are insignificant. These results show that there is no predictability from *IDP* directly, or indirectly through the original timeseries that originated it — i.e., the smoothed or actual price-earnings ratio or the orthogonal component of smoothed earnings-price ratio to *IDP*. In addition, IDP presents the value of zero during 59% of the months in our time-span. This is a very stale time-series. We repeat the previous analysis analyzing only the non-zero *IDP* months. Our conclusions remain. It is important to stress that the correlation between *IDP* and *LTM* is low, at 0.26. Accordingly, our paper shows that endogenous sectoral considerations lead to a better empirical measurement of time-varying disaster risk than a model with no such considerations.

3.3. Time-Varying Predictability

The previous section tested the in- and out-of-sample *ERP* predictability using the entire time span. There is a concern that this predictability holds only for the chosen window. Henkel, Martin, and Nardari (2011) find that traditional predictors, such as short-term interest rate (*TBILL*) and dividend yield (*DY*), have no predictive power during economic expansions in a sample of the G7 countries but do during contraction periods. Dangl and Halling (2012) find that *ERP* static-time regressions underestimate the predictive ability of some variables during particular periods of time, such as crises, which are rare disasters. They develop a time-varying regression framework under which the estimated parameters α and β of the regression in Equation (9) are time-varying: α_t and β_t . They report up to 5.8% more profits in an asset management exercise than when using static regressions.

We follow the same idea and run simple time-varying regression tests in a setup similar to backtesting. The parameters of the regressions α_t , β_t , are calculated using always the same final point, December 2013. The first starting point is January 1993, and each month, we will move one month ahead until December 2002. The first window is from January 1993 to December 2013, and the last window is from December 2002 to December 2013. This will allow

¹⁹ For convergence stability, we need more than 10 years. Thus, we set 11 years (132 data points).

us to determine the robustness of our previous results. Figure 3 presents the IS (Panel A) and OOS (Panel B) results for the best eight IS predictors. ²⁰ LTM IS R-squared fluctuates between 5% and approximately 11% and is always significant. In fact, our baseline window (January 1993 to December 2013) delivers the worse performance from all windows. Notice also that this is the most important predictor in most months, although VRP is better before 1994 and delivers similar performance after 2001. LTM OOS R-squared fluctuates between about 3% and 10%, and VRP has similar performance to LTM, with some periods above and others below. It is interesting to see the persistency of these two predictors in delivering ERP predictability. Notice that most other predictors deliver volatile R-squared IS and OOS. For example, SII delivers a OOS R-squared ranging between about -6% and 3%. The predictability stemming from LTM is resilient, positive and significant, and it seems that the static performance is a lower bound of the time-varying performance.

[Insert Figure 3 here]

3.4. Stock Return Decomposition

In this section, we analyze if the predictability of *LTM* is derived from the discount rate and/or the cash flow channels. We use the framework in Rapach et al (2016), which they import from Campbell (1991) and Campbell and Ammer (1993). They use a VAR framework to extract the cash flow and discount rate news components of stock return innovations using the log return, log dividend-price ratio, and the first three principal components extracted from 14 popular predictors of Goyal and Welch (2008). They also show that using either the first three principal components or each individual predictor yields similar qualitative results. Then they run predictive regressions of each component – expected return (ER), discount rate news (DR), and cash flow news (CF) – on *SII* and show that *SII* is relevant for future aggregated cash flows.

We follow this setting and use *LTM* instead of *SII*. Table 4 shows the estimated results for the slope of the three predictive regressions with the dependent variables given by ER, DR, and CF and the independent variable given by *LTM*. We also present results for *SII* for comparison. The ability of *LTM* to anticipate cash flow news is clearly the most economically important source of *LTM*'s predictive power for stock returns. The estimate is 0.84 and highly

²⁰ Additional predictor's time-varying regressions are provided in the online appendix.

significant with a t-statistic of 2.67. Expected return is also positive and significant, but the magnitude of the parameter is approximately one-third that of cash flow. Discount rate news presents a negligible and clearly not significant estimate. In the case of *SII*, the only driver in our sample is the cash flow component. Note that all components have opposite signs when estimating this decomposition using *LTM* versus *SII*. Most previous predictors anticipate discount rate news. As in the case of *SII*, we find that the differential information in *LTM* is relevant for future aggregated cash flows and aggregated expected returns but mainly the former.

4. Predictability in a Consumption Capital Asset Pricing Model

The equity risk premium (*ERP*) puzzle is defined according to two main hypotheses that have been tested in the literature: (i) there exists a puzzle, and thus, frictions should exist in an Arrow-Debreu equilibrium economy for the *ERP* to be as high as the empirical tests reported in the literature or (ii) there is no puzzle, and the *ERP* can be explained through the inclusion of some market-intrinsic rational distortions, such as underestimated rare disaster events. The existence of predictability of the *ERP* is independent of which hypothesis is accepted, although IS and OOS *ERP* predictability better suits the rare disaster economic models in which investors optimally hedge "times" the occurrence of a rare disaster by observing the herding effect of the cross-sectoral downturns.

In this section, we test *ERP* predictability with two of the most accepted theories to explain the *ERP* puzzle: the existence of rare disasters in a time-static framework (Barro, 2006) and in a time-varying framework (Wachter, 2013). The theoretical results of this section provide intuition for why *LTM* would predict the equity risk premium by building a stylized simple model and testing against previous models in the literature.

4.1. General framework

We consider a standard economy with a single household that represents all consumers. This household maximizes the utility of consumption $U(C_t)$ in an infinite horizon

$$E_t \left[\sum_{t=0}^{\infty} \beta^t U(C_t) \right], \tag{12}$$

where β is the time discount factor.²¹

The form of the utility function is defined within each particular asset pricing consumption-based model into which we incorporate endogenous sector risk activity. The representative agent must decide between consuming at time t or investing and waiting to consume at time t+1. Therefore, in equilibrium, the fundamental relation holds

$$P_t^e U'(C_t) = \beta E_t \left[(P_{t+1}^e + C_{t+1}) U'(C_{t+1}) \right], \tag{13}$$

where P_{t+1}^e represents the equity price at time t+1.

We introduce a multi-asset endowment economy to model the n sectors of the economy where the joint consumption process $\mathbf{C}_t = (C_{1,t}, \dots, C_{n,t})$ is a vector of each sector consumption $C_{i,t}$ in which the sectors have a weighting $\mathbf{\omega} = (\omega_1, \dots, \omega_n)$. Hence, the aggregated consumption is given by $C_{M,t} = \mathbf{\omega}^t \mathbf{C}_t$. We use the M subscript to define the aggregated variables; then, the previously defined consumption in Equation (12) is equivalent to the aggregated consumption $(C_t = C_{M,t})$. The dividend process is defined by $\mathbf{D}_t = (D_{1,t}, \dots, D_{n,t})$, where the aggregated dividend is $D_{M,t} = \mathbf{\omega}^t \mathbf{D}_t$. The equity market price $P_{M,t}^e$ is the aggregated price process of the sectors: $P_{M,t}^e = \mathbf{\omega}^t \mathbf{P}_t^e$ where $\mathbf{P}_t^e = (P_{1,t}^e, \dots, P_{n,t}^e)$ is the joint price process of the sectors, $R_{M,t}^e = \mathbf{\omega}^t \mathbf{R}_t^e$ is the corresponding aggregated equity return where $\mathbf{R}_t^e = (R_{1,t}^e, \dots, R_{n,t}^e)$, and the equity risk premium is given by $ERP_t = \mathbf{\omega}^t \mathbf{R}_t^e - R_t^b$ with R_t^b being the treasury bill rate. In the rare disaster consumption models (Rietz 1988, Barro 2006, and Wachter 2013), an additional factor that represents the consumption collapse is added to the classic equity risk premium formulation:

$$ERP_{t} = ERP_{t}^{\text{standard}} + ERP_{t}^{\text{disaster}},$$
 (14)

²¹ Lucas's (1978) assumption of a single infinitely living consumer seems unrealistic. However, we can consider an economy in which wealth is inherited by household successors with no frictions.

where ERP_t^{standard} is the Mehra and Prescott (1985) equity risk premium.²² Barro (2006) models ERP_t^{disaster} as a time-static factor that prices a low probability consumption collapse, while Wachter (2013) includes a time-varying factor in ERP_t^{disaster} that models the high volatility of stock market returns.

4.2. Endogenous Sectors in a Classical Equity Puzzle Model (No Rare Disasters)

Even though *ERP* predictability theory suits the rare disaster consumption models, predictability can be explained in the more simple case of the classic puzzle framework of Mehra and Prescott (1985). In an explanatory paper, Mehra (2003) demonstrates that the source of the equity risk premium comes from the covariance between the derivative of the utility of future consumption and equity returns:

$$ERP_{t} = ERP_{t}^{\text{ standard}} = E_{t}\left(R_{M,t+1}^{e}\right) - R_{t+1}^{f}$$

$$= \frac{1}{E\left[U'\left(C_{M,t+1}\right)\right]} \operatorname{cov}_{t}\left(-U'\left(C_{M,t+1}\right), R_{M,t+1}^{e}\right), \tag{15}$$

where

$$U\left(C_{M,t}\right) = \frac{C_{M,t}^{1-\theta}}{1-\theta},\tag{16}$$

and the joint distribution of consumption and equity prices is modeled by a bivariate log-normal distribution. The past occurrence of crises such as the one in 2007-2008 undermines this assumption, which is corroborated by the dataset collected in Barro and Ursua (2008). This assumption of a log-normal distribution is not present in the original formulation of the ERP puzzle. Therefore, we can use a different distribution to explain sectoral predictability; for example, the covariance between the marginal utility of consumption and equity prices can be the covariance of a bivariate heavy-tailed distributed, e.g., the distribution of a bivariate jump-diffusion process or a bivariate Student t.

In Figure 4, we plot the standardized *LTM* from the 10 MSCI U.S. sectors from January of 1993 to December of 2013 (as defined in Section 2) and the standardized marginal utility of

²² Mehra and Prescott (1985) establish that the source of the *ERP* is the uncertain bivariate relation between future consumption and dividends. Barro (2008), on the other hand, determines that the *ERP*'s source is the univariate randomness of the future price/dividend ratio in relation to consumption per capita.

the personal consumption core price index, $U'(C_{M,t+1})$, where the utility function is a CRRA utility with a risk aversion value of $\theta=4$. Personal consumption is used in the construction of the consumption per capita presented in the online dataset from Robert Schiller (Case and Schiller, 2003), which is frequently used in the consumption asset pricing literature. From Figure 4, we can observe a strong correlation between the *LTM* and the marginal utility $U'(C_{M,t+1})$. Both standardized marginal utility and standardized *LTM* series are detrended. We regress the standardized marginal utility on the standardized *LTM* and obtained an in-sample (IS) R-squared of 19.77%. This high correlation between the two variables is insufficient to explain the full *ERP*; a high volatility of the marginal utility $U'(C_{M,t+1})$ and/or a high volatility of the equity returns is still required to explain the *ERP* puzzle in the classic framework. However, a bivariate heavy-tailed distribution of the marginal utility and the equity returns can explain a high level of the *ERP* in the classic framework. To contrast with previous literature, we use the Treasury bill rate (*TBILL*) in the same regression. When we use the *TBILL* alone to explain marginal utility, the IS R-squared is only 7.68%. However, when tested jointly with the *LTM*, the IS R-squared is 21%. This shows the meaningful impact of using the *LTM* to predict the *ERP*.

[Insert Figure 4 here]

4.3. Endogenous Sectors in a Static Rare Disaster Model

Barro's (2006) consumption model incorporates the disaster factor that was originally proposed in Rietz (1987), but using worldwide data to calibrate the parameters. In the Barro framework, the consumption change is split into three components:

$$\log\left(\frac{C_{i,t+1}}{C_{i,t}}\right) = \gamma + x_{i,t+1}^{\text{standard}} + x_{i,t+1}^{\text{disaster}},\tag{17}$$

where γ is a constant. Notice that all production is fully consumed $\left(C_{M,t}=D_{M,t}\right)$ as in the Lucas (1978) tree. Consequently, in this model there is a perfect transmission of the equity sector interactions to the consumption sector interactions by this link equation $\left(D_{M,t}=\boldsymbol{\omega}^{T}\mathbf{D}_{t}=\boldsymbol{\omega}^{T}\mathbf{C}_{t}=C_{M,t}\right)$. We will observe a similar equity and consumption sector relationship in the next rare disaster model (Wachter, 2013). Then,

$$\log\left(\frac{C_{i,t+1}}{C_{i,t}}\right) = \log\left(\frac{D_{i,t+1}}{D_{i,t}}\right). \tag{18}$$

This model assumes a similar power utility function as that found in Equation (16). Barro (2006) assumes no parametric form of the normal times' consumption and the rare disaster times' consumption factors; nevertheless, for the purpose of comparing with the time-varying model of Wachter (2013) and being able to calculate the *LTM* we assume a jump-diffusion form for the consumption change:

$$\log\left(\frac{C_{i,t+1}}{C_{i,t}}\right) = \mu_i + \sigma_i W_{i,t} + \left(e^{K_{M,t}} - 1\right) N\left(\lambda_t\right),\tag{19}$$

where μ_i and σ_i are sector *i*'s consumption growth mean and volatility in a standard (normal) time period, and $K_{M,i}$ the aggregated disaster decline return variable with distribution κ . The standard and the disaster consumption components are defined as follows:

$$\log \left(\frac{C_{i,t+1}}{C_{i,t}}\right)^{\text{standard}} = \mu_i + \sigma_i W_{i,t},$$

$$\log \left(\frac{C_{i,t+1}}{C_{i,t}}\right)^{\text{disaster}} = \left(e^{K_{M,t}} - 1\right) N\left(\lambda_t\right).$$
(20)

where $W_{i,t}$ is a discrete Brownian motion stochastic process and $N\left(\lambda_{t}\right)$ is a discrete Poisson jump process with the probability of a rare disaster λ_{t} . For comparison purposes, we refer to the Barro (2006) rare disaster model as a static rare disaster consumption asset-pricing model. The static disaster risk responds to the aggregated disaster decline return variable, $K_{M,t}$, when a collapse shock occurs; i.e., $ERP_{t}^{\text{static disaster risk}} = f\left(K_{M,t}\right)$. The aggregated disaster decline mean return is given by $\mu^{J} = \omega^{i} \mu^{J}$ and its variance by $\left(\sigma^{J}\right)^{2} = \omega^{i} \left(\mathbf{v}^{J} \mathbf{v}^{J}\right)^{i} \omega$, where μ^{J} is the sector disaster mean return size vector and \mathbf{v}^{J} is the sector disaster variance. Note that this setting implies that during the occurrence of a disaster, the shock size is different for each sector of the economy, but all sectors perceive the start of the rare disaster at the same time.

In Barro (2006), the consumption decline has no parametric form and is empirically estimated from the data. Instead, we use the Das and Uppal (2004) multivariate jump-diffusion to

model the disaster percentage declines $\exp(K_{M,t})$ and decompose the aggregate disaster decline returns $K_{M,t}$ by sector; $K_{M,t} = \omega^{\dagger} \mathbf{K}_t$, where $\mathbf{K}_t = (K_{1,t}, ..., K_{n,t})$ is the joint distribution of the disaster contraction for each sector.^{23,24} In our empirical test, we use the distribution of the 10 U.S. sectors defined by the Standard and Poor's 500 (S&P 500).²⁵ The aggregated variable $K_{M,t}$ disseminates the dependence and tail dependence effects that can only be observed in a multivariate setting. We use a multivariate jump-diffusion to model sectoral consumption rare disasters for ease of presentation and computation of the method of moments so that we can disentangle the consumption process in normal periods from the rare disaster consumption process. Previous modeling of rare disasters with jump-diffusion include, as examples, Liu, Longstaff and Pan (2003) and Das and Uppal (2004). In the case in which only the sectoral effects in the static rare disaster premium are considered, the *ERP* is given by (see Appendix A)

$$ERP_{t} = \theta \sigma_{C}^{2} + \lambda_{t} E_{\kappa} \left[\underbrace{e^{-\theta(\mathbf{o}^{\mathsf{T}}\mathbf{K}_{t})}}_{\text{standard premium}} - 1 \right]. \tag{21}$$

The *ERP* in Equation (21) has two sources of sectoral dependence: (i) the sectoral dependence in normal periods, which affects the volatility of consumption growth, $\sigma^2 = \omega' \Sigma_t \omega = \omega' ((\sigma_t \sigma_t') \otimes \Lambda_t) \omega$ where Σ_t and Λ_t are the corresponding standard period sectoral consumption growth covariance and correlation matrices and σ_t is the standard period sectoral consumption volatility vector; and (ii) the disaster period sectoral dependence that has its source in the sectoral disaster decline return variable K_t .

²³ In the empirical tests of the model in Section 3, we assume that the sectoral equity tail dependence structure is induced by the sectoral consumption tail dependence structure. This assumption follows from Barro (2006) and Wachter (2013) using L = 1 in the link equation $D_{r} = C_{r}^{L}$ where L is the dividend leverage.

 $^{^{24}}$ $K_{M,r}$ is calibrated from the empirical dataset of Barro and Ursua (2008), and ω is given by the equity sector weights that are assumed to be equal to consumption sector weights considering the link equation $D_r = C_r^L$, and then, K_r is calculated using Das and Uppal (2004).

²⁵ In a theoretical exercise at the end of this section, we adjust the U.S. sector data to the sectors of the world economy by an observed rare disaster adjustment correlation that considers no observations of consumption disasters for the U.S. economy from January 1993 to December 2013.

4.4. Endogenous Sectors in a Time-Varying Rare Disaster Model

One main problem with the static rare disaster models is that they cannot explain *OOS ERP* predictability but only *IS ERP* predictability. One of the main findings in this paper is the significant out-of-sample predictability of *LTM* for *ERP* over the traditional *ERP* predictors. For this reason, we develop an endogenous sector time-varying rare disaster consumption model using Wachter (2013) as a baseline.

There is an endowment economy where the sector *i*-th consumption evolves according to

$$\frac{dC_{i,t}}{C_{i,t}} = \mu_i dt + \sigma_i dW_{i,t} + \left(e^{K_{i,t}} - 1\right) dN_{i,t} \left(\lambda_t\right),\tag{22}$$

where $dW_{i,t}$ is a continuous Brownian motion, $K_{i,t}$ is the sector disaster decline return, and $dN_i\left(\lambda_t\right)$ is a continuous Poisson jump process. We assume that the occurrence of consumption disasters is perfectly correlated; that is, once a disaster occurs, it affects all sectors of the economy: $\lambda_{1,t} = \lambda_{2,t} = \ldots = \lambda_{n,t} = \lambda_t$. Nevertheless, the effects on sectors are heterogeneous, as modeled with different shock sizes for each sector: sectoral shock mean size is defined as μ_i^J and sectoral shock mean volatility as σ_i^J .

The dividend process is defined as a leveraged consumption:

$$D_{M,t} = C_{M,t}^L, (23)$$

where L is the leverage. Then,

$$\frac{dD_{i,t}}{D_{t^{-}}} = \mu_i^D dt + L\sigma_i dB_{i,t} + (e^{LK_{i,t}} - 1)dN_{i,t}(\lambda_t),$$
(24)

where $\mu_i^D = L\mu_i + \frac{1}{2}L(L-1)\sigma_i^2$, and $dB_{i,t}$ is a Brownian motion.

Wachter (2013) implements two major changes in the assumptions of the static rare disaster models for solving the volatility of stocks and dividends puzzle: (i) the probability of a

²⁶ This assumption has an economic motivation: given the interdependence of the sectors of the economy, no sector can be isolated from a disaster that affects the entire economy.

disaster is stochastic rather than constant in time, which allows for time-series predictability; (ii) using a recursive Epstein and Zin (1989) utility function

$$U_t = E_t \int_t^\infty f(C_s, U_s), \tag{25}$$

where

$$f(C,U) = \beta(1-\gamma)U\left(\log C - \frac{1}{1-\gamma}\log((1-\gamma)U)\right). \tag{26}$$

allows the model to have two parameters instead of one as in the power utility case; these parameters separate risk preferences from time substitution. Consequently, the agent can select the portfolio by a time preference without affecting the risk-free asset preference through the outcome of a disaster. This second assumption will help explain the *ERP* out-of-sample predictability.

The disaster decline $1-e^{K_{MJ}}$ is adjusted to a multinomial distribution with the actual declines collected by Barro and Ursua (2008). The time-varying disaster risk present in the *ERP* is decomposed into a static disaster risk premium and a price-dividend risk premium:

$$ERP_{t} = ERP_{t}^{\text{standard}} + \underbrace{ERP_{t}^{\text{price-dividend risk}} + ERP_{t}^{\text{static disaster risk}}}_{\text{time-varying disaster risk}}.$$
(27)

Similar to the static disaster risk, the price-dividend risk responds to the aggregated disaster decline return variable, $ERP_t^{\text{price-dividend risk}} = f(K_{M,t})$. The objective of the disaster period dependence factor is to price extreme value tail-dependent events. The model in Equation (21) can be expanded for the time-varying case as

$$ERP_{t} = L\theta\sigma_{C}^{2} - \frac{1}{\text{standard premium}}$$

$$\lambda_{t} \frac{G_{t}'}{G_{t}} b\sigma_{\lambda}^{2} + \lambda_{t} E_{\kappa} \left[\underbrace{e^{-\theta(\mathbf{o}'\mathbf{K}_{t})}}_{\text{static disaster premium}} - 1 \right] \left((1 - q) \left(1 - e^{L(\mathbf{o}'\mathbf{K}_{t})} \right) + q \left(e^{(\mathbf{o}'\mathbf{K}_{t})} - e^{L(\mathbf{o}'\mathbf{K}_{t})} \right) \right) \right], \quad (28)$$

$$\underbrace{\text{static disaster premium}}_{\text{time-varying disaster premium}}$$

where G_t is a function of the price-dividend ratio and b, G_t , G_t are as described in Wachter (2013). The price-dividend ratio and the static disaster risk premium also depend on other variables: the relative risk aversion, θ , the rate of time preference, β , the average consumption

growth, μ , the volatility of consumption growth in normal periods, σ , the leverage of the consumption, L, the probability of a default by a disaster, q, and the parameters of the disaster shock, i.e., the probability of a rare disaster, λ_i , the total volatility of a rare disaster, σ_{λ}^2 , and the mean jump and volatility of the rare disaster occurrence per sector, μ^J , \mathbf{v}^J .

From Das and Uppal (2004), the total consumption growth covariance is given by

$$\operatorname{cov}\left(\frac{dC_{i,t}}{C_{i,t}}, \frac{dC_{j,t}}{C_{i,t}}\right) = t\left(\sigma_i \sigma_j \rho_{i,j} + \lambda_t \left(\mu_i^J \mu_j^J + \nu_i^J \nu_j^J\right)\right). \tag{29}$$

From Equation (28), we observe that when there is no disaster, the sector covariance is $t\Sigma_t$, but when a disaster occurs, it increases to $t(\Sigma_t + \lambda_t(\mu^J \mu^J + \mathbf{v}^J \mathbf{v}^J))$. Therefore, the endogenous sectoral correlation in normal periods is given by

$$\rho\left(\frac{dC_{i,t}}{C_{i,t}}, \frac{dC_{j,t}}{C_{j,t}}\right) = \rho_{i,j},\tag{30}$$

whereas the endogenous sectoral correlation during a disaster is

$$\rho^{J} \left(\frac{dC_{i,t}}{C_{i,t}}, \frac{dC_{j,t}}{C_{j,t}} \right) = \rho_{i,j}^{J} = \frac{\lambda_{t} \left(\mu_{i}^{J} \mu_{j}^{J} + v_{i}^{J} v_{j}^{J} \right)}{\left(\lambda_{t} \left(\left(\mu_{i}^{J} \right)^{2} + \left(v_{i}^{J} \right)^{2} \right) \right)^{1/2} \left(\lambda_{t} \left(\left(\mu_{j}^{J} \right)^{2} + \left(v_{j}^{J} \right)^{2} \right) \right)^{1/2}}, \quad (31)$$

and the total sectoral correlation (standard + disaster times) is

$$\rho^{LTM}\left(\frac{dC_{i,t}}{C_{i,t}}, \frac{dC_{j,t}}{C_{j,t}}\right) = \rho_{i,j}^{LTM} = \frac{\left(\sigma_{i}\sigma_{j}\rho_{i,j} + \lambda_{t}\left(\mu_{i}^{J}\mu_{j}^{J} + v_{i}^{J}v_{j}^{J}\right)\right)}{\left(\sigma_{i}^{2} + \lambda_{t}\left(\left(\mu_{i}^{J}\right)^{2} + \left(v_{i}^{J}\right)^{2}\right)\right)^{1/2}\left(\sigma_{j}^{2} + \lambda_{t}\left(\left(\mu_{j}^{J}\right)^{2} + \left(v_{j}^{J}\right)^{2}\right)\right)^{1/2}}.$$
 (32)

We are interested in estimating the effects of sectoral tail dependence over the *ERP* and comparing them to the aggregate tail dependence effects. The correlation in Equation (31) can be considered as an extreme correlation. Recall that the average sectoral left tail dependence, *LTM*, *is* defined as

$$LTM = \binom{n}{2}^{-1} \sum_{i \neq j}^{n} \overline{\chi}_{i,j}. \tag{33}$$

To compute the *LTM*, we use a result by Poon, Rockinger and Tawn (2004).²⁷ If the disaster percentage decline $\exp(\mathbf{K}_t)$ is multivariate log-normally distributed, then (i) the multivariate distribution of the sectoral consumption growth is multivariate log-normal, and the bivariate tail dependence of the sectors are equal to the correlation in Equation (31): $\overline{\chi}_{i,j} = \rho_{i,j}^{LTM}$, and (ii) the bivariate tail dependence of the jumps is equal to 1, i.e., $\overline{\chi}_{i,j}^J = \rho_{i,j}^J = 1$. Hence, the resulting *LTM* is an average of the tail dependence of all sectors in Equation (30):

$$LTM = \binom{n}{2}^{-1} \sum_{i \neq j}^{n} \rho_{i,j}^{LTM}, \tag{34}$$

The result from (i) and (ii) is that an increase in the disaster probability λ_t increases the effects of the sector tail dependence in the *ERP*. Thus, there is a direct and visible impact between tail dependence and asset prices.²⁸

The next step is to contrast this with the aggregated tail dependence, *ALTM*. First, recall that we define the aggregate left tail mean as

$$ALTM = E\left[\mathbf{\omega}'\mathbf{R}_{t+1}^{e}\middle|\mathbf{\omega}'\mathbf{R}_{t+1}^{e} > r_{u}\right],\tag{35}$$

where r_u is a predetermined exceedance level. Generally, r_u is considered the return over a certain percentile. Then, in the time-varying rare disaster case is given by

$$ALTM_{t} = E \left[ERP_{t} + R_{t}^{b} - r_{u} \right]^{+}$$

$$= E \left[L\theta\sigma^{2} - \lambda_{t} \frac{G_{t}'}{G_{t}} b\sigma_{\lambda}^{2} + \lambda_{t} E \left[\left(e^{-\theta K_{M,t}} - 1 \right) \left((1 - q) \left(1 - e^{LK_{M,t}} \right) + q \left(e^{K_{M,t}} - e^{LK_{M,t}} \right) \right) \right] + R_{t}^{b} - r_{u} \right]^{+}.$$

$$(36)$$

In Equation (36), we observe that the aggregate tail dependence has an inverse causal relation with the *ERP*; an increase in the *ERP* triggers an increase in the *ALTM*, but this relation is not necessarily persistent in the other direction, and other factors such as the treasury bill price

²⁷ $\overline{\chi}_{i,j}$ is mathematically defined in Section 2..

²⁸ Other asymptotic dependent tail models, such as the logistic distribution or even non-parametric tail distributions (such as copulae), can be used instead of the asymptotic independent Gaussian model with the implication of a larger LTM. However, their use will not change the dynamic relation between an increase in the disaster probability λ and the increased tail dependence LTM value and, consequently, an increase in the ERP.

or the exceedance return level r_u can reverse the positive relation between the ERP and the ALTM. Additionally, this relation only holds for the tail distribution of the ERP; that is, during normal periods, the ALTM should be close to zero and ignore small changes of the ERP. We expect to observe this behavior in the empirical tests in Section 3. Recall that another important measure in this setting is the average univariate tail dependence of each sector, the SLTM, defined as

$$SLTM = n^{-1} \sum_{i=1}^{n} E\left[\omega_{i} ' R_{i,t+1}^{e} \middle| \omega_{i} ' R_{i,t+1}^{e} > r_{i,u}\right].$$
(37)

If $r_{i,u} < r_u$ on average, then ALTM < SLTM; but if $r_{i,u} > r_u$ on average, then ALTM > SLTM. Although the SLTM incorporates part of the bivariate tail dependence effects implying better predictability for the ERP than the ALTM, the effects are only visible when each pair of sectors i, j has returns over each of their own sector tail thresholds $r_{i,u}, r_{j,u}$ ignoring the tail dependence effects of lower comovements. For this reason and by the Jensen inequality, the rank of predictability, Pred(X), of these three tail variables is expected to follow the inequalities:

$$Pred(LTM) > Pred(SLTM) > Pred(ALTM).$$
(38)

The static and the time-varying rare disaster models in Equations (21) and (28) agree with all previous asymmetric tail dependence models and empirical tests (Longin and Solnik 2001, Ang and Chen 2002, Poon, Rockinger and Tawn 2004, and Ang, Chen and Xing 2006) in which bivariate tail dependence is higher for negative returns. In the empirical section of this paper, the difference in the mean observed values of the *LTM* and the *RTM* is consistent with the model and the previous literature.

4.5. Results

To assess the impact of the *LTM* in the *ERP*, we run an asset pricing exercise computing the *ERP* for different levels of sectoral left tail dependence, *LTM*. We use the 10 sectors of the economy as defined by the S&P 500 index with the sector weights set to those of December 2013.²⁹ First,

²⁹ We additionally tested different sector weights, such as those of the S&P500 in December 2016, 2015, 2014, and 2008 and an equally weighted portfolio. The magnitude of the results of the *LTM*'s impact over the *ERP* remains unchanged.

we calibrate the consumption decline, $1-e^{K_{MJ}}$, parametric jump-diffusion distribution to the observed declines by Barro and Ursua (2008). In Figure 5, we have the resulting calibrated distribution. Additionally, we plot the empirical calibrated multinomial distribution of Wachter (2013) for comparison. There is a higher kurtosis for the nonparametric multinomial adjusted distribution, but once we truncate the log-normal distribution, it provides similar results in terms of the observed *ERP* for the different disaster probabilities λ_t .

[Insert Figure 5 here]

Second, we estimate the sectoral time-varying rare disaster jumps by calibrating the jump parameters μ^J and \mathbf{v}^J through the application of the method of moments, as presented in Das and Uppal (2004), where the correlation during "normal periods" $\rho_{i,j}$ is set to the observed unconditional correlation. This is the "normal periods" correlation because U.S. consumption did not present any consumption disasters (i.e., periods with $C_{t+1}/C_t < -10\%$) between January 1973 and December 2013. However, it is still possible to observe periods of increased distress, such as the 2007-2008 crisis. Therefore, we assume that the observed unconditional correlation has traces of a rare disaster, and we modeled this considering an absorbed rare disaster observation factor, ABS_JUMP . Then, the final observed "normal periods" correlation is given by $\rho_{i,j} \times ABS_JUMP$. In this exercise, we set $ABS_JUMP=1$ to observe the LTM. After the multi-asset model parameters are found by the method of moments, changes to the parameters of the disaster jump univariate log-normal distribution are estimated by increasing/decreasing the implicit total correlation $\rho_{i,j}^{LTM}$ (LTM = normal periods correlation + disaster tail dependence), which is directly equivalent to an increase or decrease in the LTM.

Third, we use the Wachter (2013) time-varying rare disaster model's calibrated parameters for the probability of disasters and compute the *ERP* for different values of the *LTM*. In Figure 6, we observe the resulting *ERP*. At the average disaster probability $\overline{\lambda}_i = 0.355$, we observe that LTM = 0.77 with an unconditional correlation average of $CORR = \sum_{i \neq j}^{10} \rho_{i,j} = 0.58$. This is about the average value of the *LTM*, 0.73, from the MSCI U.S. Sectors (January 1993 to

December 2013).³⁰ Moreover, this result is consistent with the ABS_JUMP of less than one, which implies that part of the rare disaster jump is observed in the normal period correlation. *Ceteris paribus*, an increase in the LTM from 0.57 to 0.77 drives an increase of 1.26% in the ERP. In practice, we have observed one-month changes of 0.10 in the LTM that should drive a direct increase of 0.63% of the ERP, which is a 23% increase in relative terms of the initial ERP of 5.56% for a LTM of 0.57. Notice that this increase in the ERP is 9 for an average disaster probability of $\overline{\lambda}_i = 0.355$. During disaster periods, we expect this probability to be higher and result in a further increase in the ERP due to the LTM.

[Insert Figure 6 here]

5. Conclusion

This paper examines the predictability of the equity risk premium by the endogenous sectors' bivariate tail dependency. We demonstrate not only that stock market predictability is not dead but also that it has economic value for investors.

A new measure of country left tail dependence is proposed, which is based on the cross-sectional left tail behavior of its pairs of sectors (*LTM*). This measure is derived by combining two types of information sets: the information in the tails and the intra-country sector relation. We give evidence of the predictability of stock market premiums using this measure in exercises in- and out-of-sample. No other variable, including the variance risk premium, can predict the stock market risk premium significantly better than the historical average. We also show that this significance is not a result of a data mining exercise. We show that the information at the industry level and their dependences in the tails are crucial for this outcome. Moreover, the new tail measure that is based on more granular information about dependency of sectors is superior to aggregate univariate tail measures. We also show that the economic source of *LTM* predictive power stems predominantly from a cash flow channel and its predictability – although time-varying – is resilient, which is not the case with all other common predictors.

A multi-asset theoretical framework is developed based on the joint endogenous sectoral comovement impact over equity prices. An explicit and economically significant relation

³⁰ This value is computed in the empirical implementation of this measure in Section 3.1.

between the increase of the probability of a rare consumption disaster and the increase in the tail dependence of the sectors of the economy is derived with implications for the asset pricing literature.

All in all, joint left tail sector relations play an important role in stock market predictability.

Appendix A. Derivations

A.1 The static and the time-varying rare disaster ERP with endogenous sectors

The derivation of the static, Equation (21), and the time-varying, Equation (28), endogenous sectors *ERP* is the result of an extension of the Wachter (2013) time-varying model (Equation 29 of Wachter 2013). In Wachter (2013), the general time-varying *ERP* model is

$$ERP_{t} = L\theta\sigma_{C}^{2} - \lambda_{t} \frac{G_{t}'}{G_{t}} b\sigma_{\lambda}^{2} + \lambda_{t} E_{\kappa} \left[\left(e^{-\theta(K_{M,t})} - 1 \right) \left((1 - q) \left(1 - e^{L(K_{M,t})} \right) + q \left(e^{(K_{M,t})} - e^{L(K_{M,t})} \right) \right) \right]$$
(39)

The univariate aggregate disaster decline return is substituted by the multivariate sectoral vector decline return's decomposition $K_{M,t} = \mathbf{\omega}^{\mathsf{T}} \mathbf{K}_{t}$, and the Equation (28) is derived. In this substitution, the model equilibrium is preserved. The use of an endogenous sector variable allows us to maintain the unitary fruit in the Lucas tree.

For the static rare disaster *ERP* version, set the leverage of the consumption as L=1 and the probability of a government default q=0. By Wachter (2013), the second term in Equation (39), $\lambda_t G_t^{\ \prime}/G_t b\sigma_\lambda^2$, is zero; then, Equation (21) is achieved.

A.2 LTM in a Consumption Capital Asset Pricing Model (CCAPM)

In the CCAPM model defined in Section 4, the sectoral consumption has a jump-diffusion distribution with the probability of a rare disaster occurring λ_i . Modeling the occurrence of a disaster by a truncated log-normal distribution enables us to use a result from Poon, Rockinger and Tawn (2004): the tail dependence measure of a normal distribution is equal to the Pearson correlation of the distribution, $\overline{\chi}_{i,j} = \rho_{i,j}^{LTM}$. Then, the *LTM* in this model can be calculated as

$$LTM = \binom{n}{2}^{-1} \sum_{i \neq j}^{n} \overline{\chi}_{i,j} = \binom{n}{2}^{-1} \sum_{i \neq j}^{n} \rho_{i,j}. \tag{40}$$

The Pearson correlation $\rho_{i,j}^{LTM}$ is barely observable with historical returns, as it includes the occurrence of consumption disasters; empirical returns are the composition of normal times plus disaster times (jump) distributions. To extract the jump component from the historical series, we

use the Das and Uppal (2004) jump decomposition, and then, we can estimate $\rho_{i,j}^{LTM}$ from the empirical return series and, thus, the *LTM*.

References

Ang, A., and G. Bekaert, 2007. Stock Return Predictability: Is It There? *The Review of Financial Studies* 20(3):651-707.

Ang, A., and J. Chen, 2002. Asymmetric Correlations of Equity Portfolios. *Journal of Financial Economics* 63(3):443-94.

Ang, A., J. Chen, and Y. Xing, 2006. Downside Risk. *The Review of Financial Studies* 19(4):1191-239.

Bae, K.-H.., G. A. Karolyi, and R. M. Stulz, 2003. A New Approach to Measuring Financial Contagion. *The Review of Financial Studies* 16(3):717-63.

Barro, R.J., 2006. Rare Disasters and Asset Markets in the Twentieth Century. *The Quarterly Journal of Economics* 121(3):823-66.

Barro, R.J., and J.F. Ursua, 2008. Macroeconomic Crises since 1870. *Brookings Papers on Economic Activity* 1:255–350.

Bollerslev, T., V. Todorov, and L. Xu, 2015. Tail Risk Premia and Return Predictability. *Journal of Financial Economics* 118:113-34.

Campbell, J.Y, 1991. A Variance Decomposition for Stock Returns, *Economic Journal* 101(405), 491-527.

Campbell, J.Y, J. Ammer, 1993. What Moves the Stock and Bond Markets? A Variance Decomposition for Long-Term Asset Returns, *The Journal of Finance* 48(1), 3-37.

Campbell, J.Y., and S.B. Thompson, 2008. Predicting the Equity Premium Out of Sample: Can Anything Beat the Historical Average? *The Review of Financial Studies* 21:1456-508.

Case, K.E. and R.J. Shiller, 2003, Is There a Bubble in the Housing Market? *Brookings Papers on Economic Activity* 2:299-342.

Chabi-Yo, F., S. Ruenzi, and F. Weigert, 2013. Crash Sensitivity and the Cross-Section of Expected Stock Returns. *SSRN Working Paper*.

Clark, T.E., and K.D. West, 2007. Approximately Normal Tests for Equal Predictive Accuracy in Nested Models. *Journal of Econometrics* 138:291-311.

Cohen, L., and A. Frazzini, 2008. Economic Links and Predictable Returns. *The Journal of Finance* 63(4):1977–2011.

Comin, D., and S. Mulani, 2009. A Theory of Growth and Volatility at the Aggregate and Firm Level. *Journal of Monetary Economics* 56(8):1023-42.

Dangl, T. and M. Halling, 2012. Predictive Regressions with Time-Varying Coefficients. *Journal of Financial Economics* 106(1):157-81.

Das, S. R. and R. Uppal, 2004. Systemic Risk and International Portfolio Choice. *The Journal of Finance* 59(6):2809-34.

Epstein, L. G. and S. E. Zin, 1989. Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: A Theoretical Framework. *Econometrica* 57(4):937-69.

Ferreira, M.A., and P. Santa-Clara, 2011. Forecasting Stock Market Returns: The Sum of the Parts is More than the Whole. *Journal of Financial Economics* 100:514-37.

Goyal, A., and I. Welch, 2008. A Comprehensive Look at the Empirical Performance of Equity Premium Prediction. *The Review of Financial Studies* 21(4):1455-1508.

Hansen, L.P., and K.J. Singleton, 1983. Stochastic Consumption, Risk Aversion, and the Temporal Behavior of Asset Returns. *Journal of Political Economy* 91(2):249-65.

Hartmann, P., S. Straetmans, and C. G. de Vries, 2004. Asset Market Linkages in Crisis Periods. *The Review of Economics and Statistics* 86(1):313-26.

Harvey, C., Y. Liu, and H. Zhu, 2016, ... and the Cross-Section of Expected Returns. *The Review of Financial Studies* 29(1):5-68.

Henkel, S. J., J.S. Martin, and F. Nardari, 2011. Time-Varying Short-Horizon Predictability. *Journal of Financial Economics* 99(3):560-80.

Hilal, S., S.-H. Poon, and J. Tawn, 2011. Hedging the Black Swan: Conditional Heteroskedasticity and Tail Dependence in S&P500 and VI. *Journal of Banking and Finance* 35(9):2374-87.

Hodrick, R., 1992. Dividend Yields and Expected Stock Returns: Alternative Procedures for Inference and Measurement. *The Review of Financial Studies* 5(3):357-86.

Holly, S., and I. Petrella, 2012. Factor Demand Linkages, Technology Shocks, and the Business Cycle. *The Review of Economics and Statistics* 94(4):948-63.

Hong, H., W. Torous, and R. Valkanov, 2007. Do Industries Lead Stock Markets? *Journal of Financial Economics* 83(2): 367-96.

Horvath, M., 2000. Sectoral Shocks and Aggregate Fluctuations. *Journal of Monetary Economics* 45(1):69-106.

Kelly, B. and H. Jiang, 2014. Tail Risk and Asset Prices. *The Review of Financial Studies* 27(10):2841-71.

Li, J., I. Tsiakas, and W. Want, 2015. Predicting Exchange Rates Out of Sample: Can Economic Fundamentals Beat the Random Walk? *The Journal of Financial Econometrics* 13:293-341.

Liu, J., F.A. Longstaff, and J. Pan, 2003. Dynamic Asset Allocation with Event Risk. *The Journal of Finance* 58(1):231-59.

Longin, F., and B. Solnik, 2001. Extreme Correlation in International Equity Markets. *The Journal of Finance* 56(2):649-76.

Lucas, R. E., 1978. Asset Prices in an Exchange Economy. *Econometrica* 46(6):14-29.

Martin, I., 2013. The Lucas Orchard. *Econometrica* 81(1):55-111.

Mehra, R., 2003. The Equity Premium: Why Is It a Puzzle? *Financial Analysts Journal* 59(1):54-69.

Mehra, R., and E.C. Prescott, 1985. The Equity Premium: A Puzzle. *Journal of Monetary Economics* 15(2):145-61.

Memmel, C., 2003. Performance Hypothesis Testing with the Sharpe Ratio, *Finance Letters* 1:21-23.

Menzly, L., and O. Ozbas, 2010. Market Segmentation and the Cross-Predictability of Returns. *The Journal of Finance* 65(4):1555–80.

Newey, W.K., and K.D. West, 1987. A Simple, Positive Semi-Definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix. *Econometrica* 55(3):703-8.

Patton, A. J., 2009. Copula-Based Models for Financial Time Series. *Handbook of Financial Time Series* 29:767-85.

Poon, S.-H., M. Rockinger, and J. Tawn, 2004. Extreme Value Dependence in Financial Markets: Diagnostics, Models, and Financial Implications. *The Review of Financial Studies* 17(2):581-610.

Rapach, D. E., M. C. Ringgenberg, and G. Zhou, 2016. Short Interest and Aggregate Stock Returns. *Journal of Financial Economics* 121(1):46-65.

Rapach, D.E., J. K. Strauss, and G. Zhou, 2010. Out-of-Sample Equity Premium Prediction: Combination Forecasts and Links to the Real Economy. *The Review of Financial Studies* 23(2): 821-62.

Rapach, D. E., J. K. Strauss, J. Tu, and G. Zhou, 2015. Industry Interdependencies and Cross-Industry Return Predictability. *Working Paper*.

Richardson, M., and T. Smith, 1991. Tests of Financial Models in the Presence of Overlapping Observations. *The Review of Financial Studies* 4(2):227-54.

Rietz, T. A. 1988. The Equity Risk Premium: A Solution. *Journal of Monetary Economics*, 22(1):117–31.

Shiller, R. J. 1989. Market Volatility. MIT Press, Cambridge, MA.

Veldkamp, L., and J. Wolfers, 2007. Aggregate Shocks or Aggregate Information? Costly Information and Business Cycle Comovement. *Journal of Monetary Economics* 54(1):37-55.

Wachter, J. A., 2013. Can Time-Varying Risk of Rare Disasters Explain Aggregate Stock Market Volatility? *The Journal of Finance* 68(3):987-1035.

Table 1 – Summary statistics

This table reports summary statistics for the dependence conditioning variables (Panel A), the traditional conditioning variables (Panel B), and the equity risk premium (Panel C). The dependence variables are CORR (correlation sectors' mean), RTM (bivariate right tail sectors' mean), LTM (bivariate left tail sectors' mean), ALTM (univariate left tail of the market), and SLTM (univariate left tail sectors' mean). All these variables are defined in Section 2.1. The traditional conditioning variables are DFS (default spread), TMS (term spread), DP (dividend-price ratio), TBILL (detrended T-bill rate), BM (book-to-market ratio), DY (dividend yield), DE (dividend payout ratio), EP (earnings-price ratio), SV (realized stock variance), NTIS (net equity expansion), INFL (inflation), LTY (long-term yield), VRP (variance risk premium), CSTR (cross-sectional tail risk), and SII (short interest index). The VRP is from Hao Zhou's website, and SII if from David Rapach's website. CSTR is computed by the authors. The remaining variables are from Amit Goyal's website. The variables are defined in Section 2.2. The assets used are the stock market returns, the long-term bond returns, and short-term bond returns. For each variable, the time series average (Mean), standard deviation (Std Dev), skewness (Skew), excess kurtosis (Kurt), minimum (Min), maximum (Max), and first-order autocorrelation ($\rho(1)$) are reported. The sample period is from January 1993 to December 2013.

	Mean	Std Dev	Skew	Kurt	Min	Max	ρ(1)
Panel A: Depende	nce cond	itioning va	riables				
CORR	0.61	0.05	0.45	-0.84	0.53	0.72	1.00
RTM	0.67	0.09	-0.52	-1.09	0.50	0.82	0.99
LTM	0.82	0.05	-1.56	2.03	0.67	0.89	0.97
ALTM	0.37	0.03	0.16	-1.12	0.32	0.45	0.98
SLTM	0.35	0.02	-0.08	-1.27	0.31	0.38	0.99
Panel B: Tradition	al condit	ioning varia	ables				
DFS (%)	1.04	0.41	2.40	9.13	0.55	3.38	0.96
TMS (%)	0.43	3.03	0.17	1.81	-11.25	14.41	0.02
DP	-1.64	0.16	0.09	-0.97	-1.96	-1.31	0.99
TBILL (%)	-0.17	0.82	-0.37	0.18	-2.75	1.71	0.96
BM	0.35	0.16	1.10	0.96	0.12	0.91	0.99
DY	-1.64	0.16	0.08	-0.94	-1.97	-1.31	0.99
DE	-0.34	0.17	2.63	10.50	-0.54	0.60	0.98
EP	-1.30	0.18	-1.48	4.29	-2.10	-0.97	0.98
SV (%)	0.28	0.57	8.23	83.74	0.02	7.09	0.45
NTIS	0.01	0.02	-0.47	0.05	-0.06	0.05	0.98
INFL (%)	0.23	0.31	-1.14	6.90	-1.92	1.22	0.46
LTY (%)	6.57	2.44	0.59	-0.02	2.06	13.81	0.99
VRP (%)	17.97	20.07	-2.41	35.35	-180.68	116.52	0.27
CSTR	0.46	0.03	-0.35	0.67	0.33	0.56	0.74
SII	3.49	1.84	0.46	-0.56	1.04	8.92	0.99
Panel C: Equity ris	sk premi	um					
ERP	0.38	4.43	-0.90	1.79	-19.16	10.60	0.10

Table 2 – Correlation matrix of conditioning variables

This table reports the unconditional correlation matrix for 20 variables. The non-dependence variables are *DFS* (default spread), *TMS* (term spread), *DP* (dividend-price ratio), *TBILL* (detrended T-bill rate), *BM* (book-to-market ratio), *DY* (dividend yield), *DE* (dividend payout ratio), *EP* (earnings-price ratio), *SV* (realized stock variance), *NTIS* (net equity expansion), *INFL* (inflation), *LTY* (long-term yield), *VRP* (variance risk premium), *CSTR* (cross-sectional tail risk), and *SII* (short interest index). The *VRP* is from Hao Zhou's website, and *SII* is from David Rapach's website. *CSTR* is computed by the authors. The remaining variables are from Amit Goyal's website. These variables are defined in Section 3.2. The dependence variables are *CORR* (correlation sectors' mean), *RTM* (bivariate right tail sectors' mean), *LTM* (bivariate left tail sectors' mean), *ALTM* (univariate left tail of the market), and *SLTM* (univariate left tail sectors' mean). The variables are defined in Section 2.1. The sample period is from January 1993 to December 2013.

	DFS	TMS	DP	TBILL	BM	DY	DE	EP	SV	NTIS	INFL	LTY	VRP	CSTR	SII	CORR	RTM	LTM	ALTM	SLTM
DFS		0.34	0.33	-0.42	0.30	0.31	0.66	-0.54	0.61	-0.58	-0.24	-0.51	0.03	-0.06	0.16	-0.39	-0.17	-0.23	0.37	0.42
TMS	0.34		0.41	-0.25	0.38	0.40	0.42	-0.22	0.12	0.17	-0.04	-0.17	0.00	0.26	-0.09	0.04	0.17	0.32	0.29	0.50
DP	0.33	0.41		0.04	0.67	0.99	0.48	0.10	0.15	-0.05	-0.10	-0.02	-0.12	0.37	0.02	0.39	0.24	0.47	0.02	0.15
TBILL	-0.42	-0.25	0.04		0.24	0.06	-0.36	0.43	-0.34	0.36	0.09	0.19	-0.10	0.04	-0.24	0.30	0.09	0.40	0.03	-0.03
BM	0.30	0.38	0.67	0.24		0.66	0.03	0.40	0.06	-0.09	-0.04	-0.47	-0.17	0.40	-0.04	0.06	0.04	0.37	0.35	0.59
DY	0.31	0.40	0.99	0.06	0.66		0.47	0.10	0.08	-0.03	-0.09	-0.02	-0.12	0.45	0.00	0.42	0.26	0.50	0.01	0.15
DE	0.66	0.42	0.48	-0.36	0.03	0.47		-0.83	0.35	-0.14	-0.09	0.03	0.19	-0.02	0.09	0.06	0.11	0.07	0.11	0.08
EP	-0.54	-0.22	0.10	0.43	0.40	0.10	-0.83		-0.30	0.13	0.04	-0.04	-0.29	0.27	-0.08	0.18	0.02	0.22	-0.11	0.01
SV	0.61	0.12	0.15	-0.34	0.06	0.08	0.35	-0.30		-0.35	-0.36	-0.17	-0.31	-0.30	0.11	-0.21	-0.14	-0.18	0.10	0.05
NTIS	-0.58	0.17	-0.05	0.36	-0.09	-0.03	-0.14	0.13	-0.35		0.06	0.56	0.17	0.08	-0.38	0.63	0.47	0.71	-0.28	-0.21
INFL	-0.24	-0.04	-0.10	0.09	-0.04	-0.09	-0.09	0.04	-0.36	0.06		0.12	0.10	-0.05	0.07	-0.02	-0.06	-0.08	0.00	-0.07
LTY	-0.51	-0.17	-0.02	0.19	-0.47	-0.02	0.03	-0.04	-0.17	0.56	0.12		-0.01	-0.19	0.06	0.51	0.16	0.27	-0.43	-0.77
VRP	0.03	0.00	-0.12	-0.10	-0.17	-0.12	0.19	-0.29	-0.31	0.17	0.10	-0.01		-0.05	-0.11	0.04	0.17	0.03	-0.13	-0.08
CSTR	-0.06	0.26	0.37	0.04	0.40	0.45	-0.02	0.27	-0.30	0.08	-0.05	-0.19	-0.05		-0.07	0.19	0.15	0.37	0.01	0.27
SII	0.16	-0.09	0.02	-0.24	-0.04	0.00	0.09	-0.08	0.11	-0.38	0.07	0.06	-0.11	-0.07		-0.59	-0.81	-0.41	0.56	0.00
CORR	-0.39	0.04	0.39	0.30	0.06	0.42	0.06	0.18	-0.21	0.63	-0.02	0.51	0.04	0.19	-0.59		0.82	0.75	-0.71	-0.43
RTM	-0.17	0.17	0.24	0.09	0.04	0.26	0.11	0.02	-0.14	0.47	-0.06	0.16	0.17	0.15	-0.81	0.82		0.59	-0.70	-0.15
LTM	-0.23	0.32	0.47	0.40	0.37	0.50	0.07	0.22	-0.18	0.71	-0.08	0.27	0.03	0.37	-0.41	0.75	0.59		-0.19	0.06
ALTM	0.37	0.29	0.02	0.03	0.35	0.01	0.11	-0.11	0.10	-0.28	0.00	-0.43	-0.13	0.01	0.56	-0.71	-0.70	-0.19		0.72
SLTM	0.42	0.50	0.15	-0.03	0.59	0.15	0.08	0.01	0.05	-0.21	-0.07	-0.77	-0.08	0.27	0.00	-0.43	-0.15	0.06	0.72	

Table 3 – Predictability

This table presents the R-squared of in- and out-of-sample predictive regressions of one single variable of the risk premium for the next month using several common predictors. Panel A presents the results for common variables and dependence variables. The non-dependence variables are *DFS* (default spread), *TMS* (term spread), *DP* (dividend-price ratio), *TBILL* (detrended T-bill rate), *BM* (book-to-market ratio), *DY* (dividend yield), *DE* (dividend payout ratio), *EP* (earnings-price ratio), *SV* (realized stock variance), *NTIS* (net equity expansion), *INFL* (inflation), *LTY* (long-term yield), *VRP* (variance risk premium), *CSTR* (cross-sectional tail risk), and *SII* (short interest index). The *VRP* is from Hao Zhou's website, and *SII* is from David Rapach's website. *CSTR* is computed by the authors. The variables are defined in Section 2.2. The dependence variables are *CORR* (correlation sectors' mean), *RTM* (bivariate right tail sectors' mean), and *LTM* (bivariate left tail sectors' mean). Panel B presents the results for different versions of *LTM*, including *ALTM* (univariate left tail of the market) and *SLTM* (univariate left tail sectors' mean). All these variables are defined in Section 2.1. The time span is from January 1993 to December 2013. The stars represent the statistically significant predictors at a 5% significance level.

Panel A. Predictability by traditional and dependence variables

	DFS	TMS	DP	TBILL	BM	DY	DE	EP	SV	NTIS	INFL	LTY	VRP	CSTR	SII	CORR	RTM	LTM
IS \mathbb{R}^2 (%)	0.53	0.03	1.18	1.54*	0.40	1.56	0.00	0.42	2.73*	1.04	0.05	0.11	6.13*	1.62*	1.96*	2.56*	1.81*	4.91*
OOS R ² (%)	-3.73	-1.76	-0.52	0.45	-1.03	-0.28	-4.16	-3.09	-2.56	-1.53	-1.58	-1.99	4.79*	0.39	0.51	0.28	-0.32	2.94*

Panel B. Predictability by different versions of the LTM

	Bivariate sector left tail average											VW		LTM			
	ALTM	SLTM	Oil	BM	IND	CG	HC	CS	TLC	FIN	TECH	UTIL	LTM	FF5	FF10	FF17	FF38
IS R ² (%)	0.15	0.17	0.54	2.44*	1.58*	0.63	4.34*	5.18*	2.24*	5.30*	0.37	2.31*	5.21*	0.38*	5.61*	6.67*	6.75*
OOS R ² (%)	-2.57	-1.86	-1.30	0.11	-0.78	-1.84	2.20*	3.18*	0.73	3.20*	-1.21	-0.46	3.98*	-2.32	3.24*	5.11*	5.14*

Table 4 – Decomposition into cash flow and discount rate components

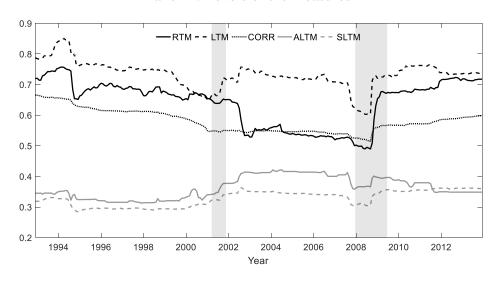
This table reports the ordinary least squares estimate of the slope of a predictive regression of expected returns (ER), cash flow news (CF), and discount rate news (DR) on the lagged variable presented in the left column, *LTM*, *VRP*, or *SII*. These three components (ER, CF, and DR) are estimated using the Campbell (1991) and Campbell and Ammer (1993) vector autoregression (VAR) framework comprising log returns, the log dividend price-ratio, and the first three principal components extracted from 14 popular predictors from Goyal and Welch (2008). Below each slope estimate, we report the heteroskedasticity- and autocorrelation-robust t-statistics. *,**,*** indicate significance at the 10%, 5%, and 1% levels, respectively. The sample period is from January 1993 to December 2013.

		ER	CF	DR
LTM	Slope	0.32 ***	0.84 ***	0.00
	t-stat	5.00	2.67	-0.01
VRP	Slope	-0.05	0.92 ***	-0.17
	t-stat	-0.59	4.21	-0.64
SII	Slope	-0.03	-0.54 ***	0.05
	t-stat	-0.78	-2.11	0.52

Figure 1 – Evolution of the dependence measures

This figure presents the evolution of the dependence measures *RTM* (bivariate right tail sectors' mean), *LTM* (bivariate left tail sectors' mean), *CORR* (sectors' correlation mean), *ALTM* (univariate left tail of the market), and *SLTM* (univariate left tail sectors' mean). Panel A presents the levels. Panel B presents the standardized variables. The standardization is performed using the unconditional moments. The gray vertical bands indicate the NBER-defined recessionary periods.

Panel A. Levels of the measures



Panel B. Standardized measures

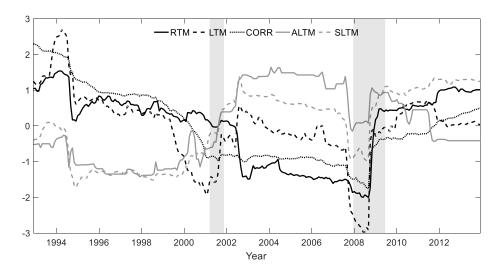


Figure 2 – Incremental value of the *LTM*

This figure presents the IS R-squared (horizontal axis) and the OOS R-squared (vertical axis) for univariate and bivariate predictive regressions. The bivariate regressions combine each variable of the univariate predictive regressions with the *LTM*. The time span is from January 1993 to December 2013.

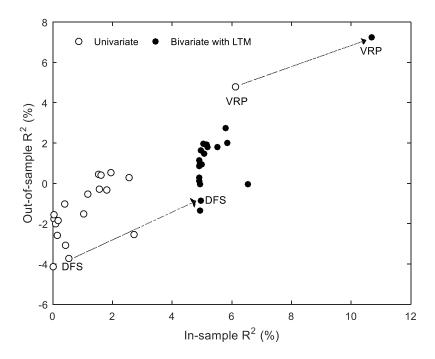
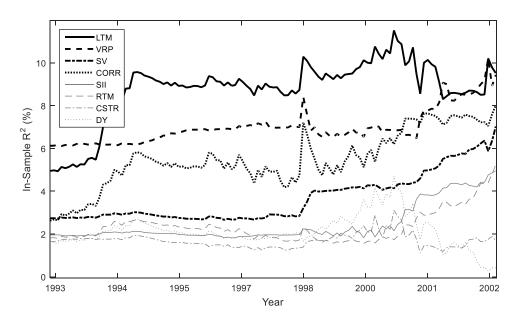


Figure 3 – Time-varying predictability

This figure presents the R-squared of in- and out-of-sample time-varying *ERP* predictive regressions for a single variable. Each point in the graphs represents the value of the R-squared for a time-varying window with the same final point (December 2013) and an initial point corresponding to that month. The results for the best eight IS predictors are presented.

Panel A. In-sample



Panel B. Out-of-sample

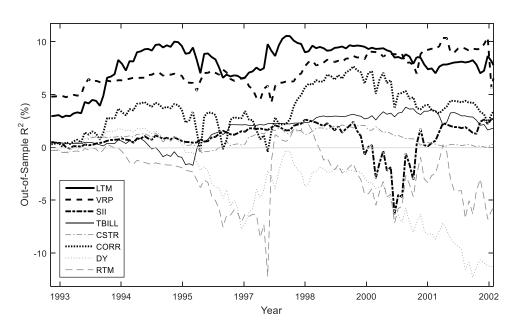


Figure 4 – The LTM of U.S. economic sectors and the marginal utility of U.S. personal consumption core price index

This figure presents the standardized *LTM* of U.S. economic sectors and the standardized marginal utility of personal consumption core price index between January 1993 and December 2013. A CRRA utility function is used with a risk aversion parameter of 4.

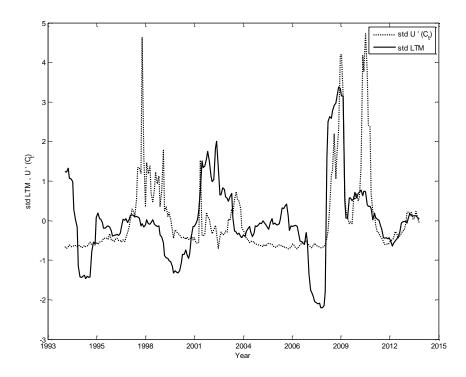


Figure 5 – Distribution of the consumption disasters of a sectoral time-varying rare disaster model

This figure presents the consumption disaster distribution for two models: (i) the time-varying rare disaster model of Wachter (2013) and (ii) a multi-asset time-varying rare disaster model. The latter uses a multivariate log-normal distribution to model joint disaster events, and it is calibrated with the multinomial distribution of the disaster contractions, as in Barro and Ursua (2008). The bars of the histogram of the multinomial consumption disaster are standardized to make the area under the bars into a probability density function.

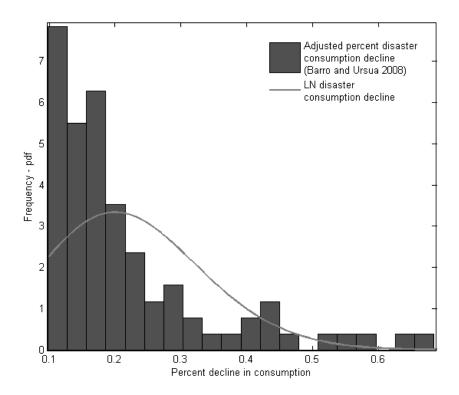
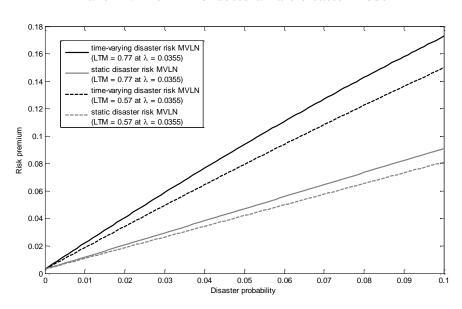


Figure 6 – The impact of LTM changes in the ERP in an endogenous sectoral rare disaster model

This figure presents the instantaneous equity risk premium from two models: (i) the time-varying rare disaster model of Wachter (2013) and (ii) a sectoral static rare disaster model as in Barro (2006). The multi-asset time-varying rare disaster model uses a multivariate log-normal distribution for modeling joint disaster events, and it is calibrated with the multinomial distribution of the disaster contractions, as in Barro and Ursua (2008). Panel A presents the static and dynamic disaster risk for different levels of tail dependence (*LTM*) for the multi-asset model. Panel B presents the monotone increasing relation of the *LTM* with the disaster probability in the sectoral time-varying rare disaster model.



Panel A. The *ERP* of sectoral rare disaster model

